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A LOWER BOUND HEURISTIC FOR SELECTING MULTI-ITEM INVENTORIES FOR A GENERAL JOB COMPLETION OBJECTIVE

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Abstract

When an inventory of multiple items is used to support the completion of an overall job or mission, commonly used inventory service level measures which are defined in terms of item availability are not appropriate. This paper develops a general model for problems of this type in which item demands are also interdependent. A heuristic is derived for determining stock levels that are guaranteed to satisfy the given inventory system performance objective, but at a slightly higher cost than would be achieved by the true optimum.

This paper describes work in progress and should not be cited without the authors' consent. Comments are welcome.

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1. Introduction

Service levels of multi-item inventory systems are often defined in terms of general performance measures, such as the number of "jobs completed" or the number of "end products" produced, where each job or end product requires a combination of the basic inventory items for completion. For example, repairing a machine in the field may require several parts and tools, any one of which is sufficient to cause an incomplete job if it is not carried in the repair service kit. In this context, the performance of the repair kit is defined as the fraction of jobs for which it allows completion, rather than the fraction of part demands that it is able to satisfy. In the context of field repair systems, several stocking analysis methodologies have been developed for this problem. [Smith, Chambers and Shlifer(1980), Mamer and Smith(1982),(1985), Graves(1982), Hausman(1982), Schaefer(1983), March and Scudder(1984)]. These approaches recognize the interdependencies between the item demands in multi-item systems, but generally restrict item stock levels to be 0 or 1. This restriction, which is imposed to obtain computationally feasible solution methods, limits the models' application to items which are restocked after each job or to items such as tools, whose inventories are not depleted after use.

In recent articles, Baker, Magazine and Nuttle(1986) and Baker(1985) analyze a structurally similar problem in the manufacturing context. Their papers consider a production system in which components are obtained in advance for uncertain demand. Later these components are assembled on order into final products. The service level of the system is defined as the probability of meeting all final product orders, while inventory

investment is determined by the stock levels of the component items. These authors analyze examples to illustrate the effect of component commonality on the cost of meeting specified final product service level targets. Despite the structural similarity between the manufacturing and field service problems, the previously discussed repair kit optimization techniques do not apply to the manufacturing problem as stated, because the item stock levels are greater than one and the component inventories are gradually depleted as end product orders are filled. Baker, Magazine and Nuttle note the complexity of solving general problems of this type and the need for more research in multi-item, multi-level inventory systems.

In this paper, we propose a heuristic for extending the previously developed job completion inventory models to the case of general stock levels with inventory depletion. Because of the complexity of the problem, we believe that a direct solution even for moderately large numbers of items and jobs (or final products in the case of manufacturing) is not computationally feasible. Instead we develop an objective function that serves as a lower bound function for the service level that is achieved for a given stock level. The minimum cost stock levels can then be found that will bring the lower bound up to a specified service level target. This approach guarantees that the service level objective will be met but, because of the use of the lower bound will tend to result in additional inventory cost. Alternatively, for a fixed budget constraint, we can maximize the service level bound. This second case is somewhat more powerful, because the resulting stock level choice can then be plugged into the actual objective function, which could be evaluated by Monte Carlo. This second case is illustrated by an example, and other intermediate results are illustrated as well.

2. Model Specifications

Key data for the problem we consider is contained in the "job matrix" J, which is defined as follows:

> J_{ij} = the number of items of type i required to complete a job (or satisfy an order) of type j.

The structure of a job matrix is illustrated in Figure 2.1 on the next page, which will serve an example for subsequent illustrative calculations. In general, we will assume there are n item types (i =

$1,\ldots,n)$ and m job types $(j = 1,\ldots,m)$.

Jobs or end product orders are assumed to form a sequence of independent events, whose interarrival times have mean $1/\lambda$. That is,

λ = rate per unit time at which requests or jobs arrive.

	0	1	2	0	4	1
	1	0	1	0	0	1
	1	1	1	0	0	0
	2	0	2	1	0	0
	1	1	0	0	1	1
Item	4	0	4	2	0	0
type i	0	1	0	2	0	1
	4	0	0	0	0	0
	0	2	0	1	2	0
	0	0	0	0	4	0

job type j

J_{ij} = # items of type i required for job j

Figure 2.1. A Job Matrix J

Job types on successive arrivals are assumed to be independent. The marginal probabilities for job types are defined as follows:

p_j = P(an arriving job or request is of type j).

In some cases, we will assume that the job arrivals are also Poisson processes.

The expected demand rates for individual items are determined by:

$$\lambda_i = \lambda \sum_j J_{ij} p_j, \tag{2.1}$$

where

λ_i = average demand rate per unit time for item i, $i=1,\ldots n$.

For any fixed time t, let us define the random variables:

$N_j(t)$ = number of requests (jobs) of type j arriving by time t, $j = 1,\ldots,m$.

$X_i(t)$ = number of items of type i requested by time t, $i = 1,\ldots,n$.

Letting

$$X(t) = X_1(t),\ \ldots,\ X_n(t)$$
$$N(t) = N_1(t),\ \ldots,\ N_m(t),$$

we clearly have the matrix relationship

$$X(t) = JN(t). \tag{2.2}$$

For a given vector $s = s_1,\ldots,s_n$ of stock levels,

s_i = initial stock level for item type i,

we also have

$$P\{\textit{all jobs completed up to time } t\} = P\{X(t) \le s\}. \tag{2.3}$$

Equation (2.3) thus defines one performance measure for the multi-item system with stock level s; namely the probability that a time t will elapse with no unsatisfied requests.

Another performance criterion of interest is:

$$E[\text{time before a stockout occurs}] = \tau$$

$$= \int_0^\infty [1 - P\{X(t) \le s\}]dt. \tag{2.4}$$

The evaluation of the inventory system's performance can be done exactly if the probability $P\{X(t) \le s\}$ in Equation (2.3) can be calculated.

This calculation is more difficult than it may first appear, however. For example, suppose that the job arrivals are independent Poisson processes.

Then $N(t)$ and $X(t)$ are clearly vector valued Markov processes, which have renewal points whenever complete restocking occurs. We are concerned with the probability distribution of the time to absorption, which occurs whenever $X_i(t) > s_i$ for some i. In principle, the steady state probabilities and the expected time to absorption can be determined analytically for these Markov processes. However, in many practical problems, the number of possible states makes the transition matrix for this problem prohibitively large. For example, if there are twenty different jobs that may be encountered and there is enough inventory so that each job may be served up to four times, the total number of different states for the vector $N(t)$ is $20^5 = 3.2$ Million. Thus only in very small problems will the exact solution of the Markov process be practical.

Monte Carlo Approximation

An alternative approach for calculating the probability $P\{X(t) \le s\}$ is Monte Carlo. This is considerably more effective from a computational standpoint than exact calculation and the number of samples to obtain any desired level of accuracy can be determined. For a given job matrix J and stock level s, let us define the indicator random variable

$$Z_k = \begin{cases} 1 & \text{if } X(t) \le s \\ 0 & \text{otherwise,} \end{cases}$$

corresponding to the k^{th} Monte Carlo sample. Then clearly

$$P\{X(t) \le s\} = \lim_{n\to\infty} S_n = (1/n)\sum_{k=1}^{n} Z_k. \tag{2.5}$$

Also, since the variance of the probability estimate S_n is maximized when the probability is $1/2$, we see that

$$\mathrm{Var}(S_n) = (1/n)P\{X(t)\le s\}[1 - P\{X(t)\le s\}] \le 1/4n.$$

Thus to obtain an accuracy of roughly two standard deviations (95%) on the second decimal place of $P\{X(t) \le s\}$, one would need to take n such that

$$2/(\sqrt{4n}) \le 0.01 \quad \text{or} \quad n \ge 10{,}000.$$

This number of simulation samples is feasible for fairly large problems, and often less accuracy is acceptable.

The main difficulty with Monte Carlo is that it provides only a point estimate and gives no indication of how the performance varies as a function of changes in stock level. This sensitivity estimation is crucial for inventory policy selection and optimization. In Section 3, we will derive a lower bound objective function that is simpler to calculate. It also provides the derivative information necessary for optimization.

The Effects of Item Demand Dependencies

In multi-item inventory control systems, a common assumption is that part demands are independent Poisson arrivals, with their arrival rates determined by the total item demands of each type observed over a period of time. It is interesting to consider the effects of simply ignoring the interdependencies between the demands for items and analyzing the inventory system as though it had independent item demands. To illustrate this, let us consider the example in Figure 2.1. Let us assume that jobs are Poisson arrivals with rate $\lambda = 1$, and all jobs are equally likely. In this case, the demand rates λ_i below would be observed for each of the items i, based on (2.1).

$$p_j = 1/6, \quad j = 1, 2, 3, 4, 5, 6$$

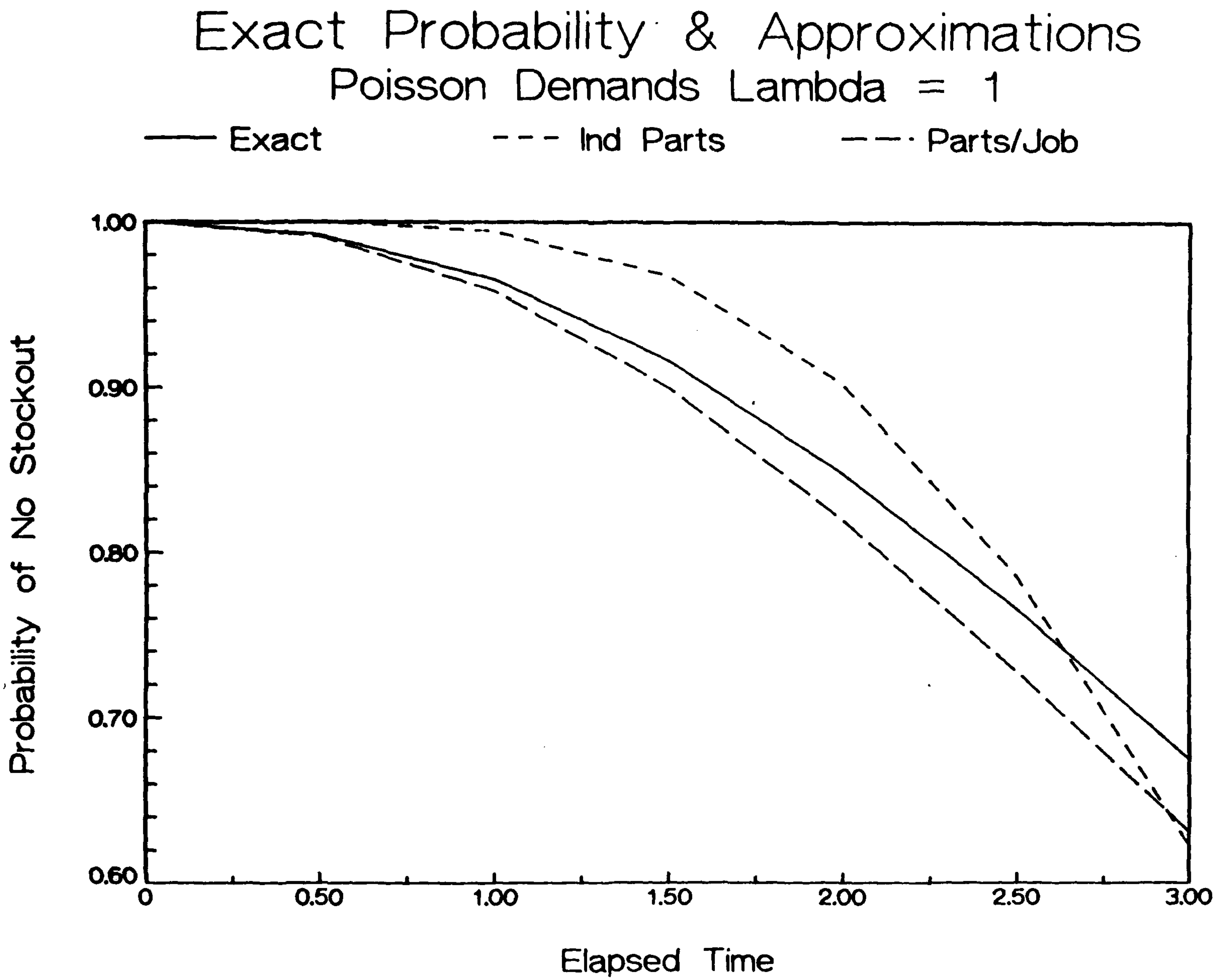
Item i	1	2	3	4	5	6	7	8	9	10
λ_i	1.17	0.5	0.5	0.83	0.67	1.67	0.67	0.67	0.83	0.67

We will determine the probability of no stockout up to time t using the independent item demand approach and the true demand process and compare the results. In this case, the problem is small enough that an accurate estimate of $P(X(t) \le s)$ can easily be determined by Monte Carlo.

The results for the stock levels

$$s = 8\ 3\ 3\ 5\ 4\ 10\ 4\ 4\ 5\ 4$$

are shown graphically in Figure 2.2. [This stock level was chosen to be twice the expected demand rate per unit time for each item type.] The "Exact" probability is $P(X(t) \le s)$ evaluated by Monte Carlo. "Ind Parts"


Exact Probability & Approximations
Poisson Demands Lambda = 1
Exact
Ind Parts
Parts/Job
Probability of No Stockout
1.00
0.90
0.80
0.70
0.60
0
0.50
1.00
1.50
2.00
2.50
3.00
Elapsed Time

is the probability determined by the product of simple Poisson probabilities that the demand for part i is less than or equal to s_i, with demand rates λ_i determined from (2.1). The third line in Figure 2.2, labeled "Parts/Job" is based on a bound, which we describe in the next section.

It is interesting to note in Figure 2.2 that the probability of no stockout calculated by the simple Poisson assumption is higher than the Exact probability of no stockout for the smaller values of t, while for larger t values it becomes less than the Exact value. The errors in both these regions are clearly significant. Thus the interdependencies in the part demands clearly cannot be ignored in many problems.

The "Parts/Job" bound has two advantages over the "Ind Parts" approximation. First, in the critical region above 90%, it is much more accurate. Second, it always lies below the true probability. Thus, if sufficient inventory is provided to meet the service level constraint calculated form the lower bound, we can be certain that the true service level has been met as well.

3. Obtaining Bounds

In this section we derive the "Parts/Job" bound in Figure 2.2. Since the exact calculations of the probability distribution of the time to stockout are complex and the sensitivity to stock level changes is even more difficult to obtain, we seek an approximation for the objective function that makes it easy to evaluate and also provides derivative information.

Let us consider the arrival process of repair jobs or demands for service, which is defined by the m independent Markov processes $N_j(t)$. We use the theory of associated random variables to obtain a lower bound for the probability of completing all jobs up to time t.

__Theorem 1.__ If the $N_j(t)$ are independent Markov processes, then we have the inequalities

$$1 - \prod_{i=1}^{n}[1-P(X_i(t) \le s_i)] \ge P(X(t) \le s) \ge \prod_{i=1}^{n} P(X_i(t) \le s_i). \tag{3.1}$$

Proof: We show that if the $\{N_j(t)\}$ are independent processes, then for each t, the random variables $\{X_i(t)\}$ are associated. Since the $N_j(t)$ are independent, they are associated random variables. Since $J_{ij} \geq 0$ for all i,j it follows that the $\{X_i(t)\}$ are nondecreasing functions of the variables $\{N_j(t)\}$. Therefore, the $\{X_i(t)\}$ are associated random variables themselves. For associated random variables, the inequalities in (3.1) are known to hold. QED.

[See Barlow and Proschan (1975), pp. 29-34 for the development of the results used above.]

We can now focus our attention on the calculation of the probability distributions of the separate processes $\{X_i(t)\}$. This in effect substitutes the calculation of m one dimensional probability evaluations for one m dimensional probability evaluation, which is in general a great simplification.

For one particular case of interest, the evaluation of the individual $\{X_i(t)\}$ results in a simple closed form solution. If the job matrix J contains only 0's and 1's , that is, no more than one of each part type is required per job and the job arrivals are independent Poisson events, we can express $P(X_i(t) \leq s_i)$ in closed form. This is based on the observation that the part demands themselves are Poisson arrivals in this case. That is, part i is demanded at time t if and only if a job arrives at time t, (a Poisson arrival) and the job is one for which $J_{ij} = 1$. This creates a Poisson process with rate

$$\lambda_i = \lambda \sum_j J_{ij} p_j .$$

Thus we have proved the following result.

Corollary 1. If the $N_j(t)$ are independent Poisson processes and the maximum entry in J is 1, then we have

$$P(X_i(t) \leq s_i) = \sum_{k=0}^{s_i} e^{-\lambda_i t} (\lambda_i t)^k / k! . \tag{3.2}$$

Furthermore, by taking the product of the $P(X_i(t) \leq s_i)$ and integrating

over t, we obtain a lower bound for the expected time to first stockout.

$$\tau \geq \sum_{k_1=0}^{s_1} \cdots \sum_{k_n=0}^{s_n} \binom{k}{k_1 \cdots k_n} (1/\lambda) \prod_{i=1}^{n} (q_i)^{k_i} \tag{3.3}$$

where $q_i = \lambda_i/\lambda$ and $k = \sum_i k_i$.

Under the assumptions of Corollary 1, the calculation of $P\{X_i(t) \leq s_i\}$ reduces to the simple "Parts/Job" method discussed previously. Thus, in this case, the simple part fill approach provides a lower bound on the system performance.

Calculating the Distribution of $X_i(t)$

In the general case, each row in the job matrix would determine the distribution of the number of parts of type i required when a demand arrives. That is, we define the random variable

$D_i(k)$ = the number of items of type i required on the k^{th} demand.

Its probability distribution is clearly given by

$$P\{D_i(k) = J_{ij}\} = p_j, \quad j = 1,\ldots,m \text{ and all } k. \tag{3.4}$$

In the Poisson arrival case, this would mean that the part demands are independent arrivals with a compound Poisson distribution. The random variable $X_i(t)$ can be expressed as

$$X_i(t) = \sum_{k=1}^{M(t)} D_i(k), \tag{3.5}$$

where $M(t) = \sum_j N_j(t)$.

In the general case, we can calculate $P\{X_i(t) \leq s_i\}$ as follows. First define

$$d_i(K) = \sum_{k=0}^{K} D_i(k). \tag{3.6}$$

Then we use the expansion formula

$$P\{X_i(t) \le s_i\} = \sum_{K=0}^{\infty} P\{d_i(K) \le s_i\} P\{M(t) = K\}. \tag{3.7}$$

Then define

$$F_i(K|s_i) = P\{d_i(K) \le s_i\}, \tag{3.8}$$

that is, the probability that part i does not stock out after K jobs. In the general case $F_i(K|s_i)$ is a multinomial probability, which must be summed over a particular set of vectors of arriving jobs. That is, we define the set of vectors

$$S_i(K|s_i) = \{(n_1,\dots,n_m) \mid \sum_j n_j = K \text{ and } \sum_j n_j J_{ij} \le s_i\}. \tag{3.9}$$

Then we have

$$F_i(K|s_i) = \sum_{n \in S_i(K|s_i)} \binom{K}{n_1 \dots n_m} \prod_j (p_j)^{n_j} . \tag{3.10}$$

Once the $F_i(K|s_i)$ are obtained, $P\{X_i(t) \le s_i\}$ is calculated from the expansion formula. It should be noted that although the sum is infinite, the terms generally become small quite quickly. For example, if the jobs are Poisson arrivals, the size of the error from dropping the remaining terms can be bounded by the tail area of the remaining Poisson terms times the last $F_i(K|s_i)$, because $F_i(K|s_i)$ is clearly decreasing in K. It is also interesting to note that truncating the expansion formula results in obtaining a job completion probability that is too small, so that the error from the approximation always results in a conservative bound.

In the case for which J_{ij} consists of only 0's and 1's, $F_i(K|s_i)$ can be expressed more simply. As before, we let q_i equal the probability

that part i is demanded on a given job,

$$q_i = \sum_j J_{ij} \; p_j = \lambda_i/\lambda.$$

Then

$$F_i(K|s_i) = \sum_{m=0}^{s_i} \binom{K}{m} q_i^{\,m} \; (1 - q_i)^{K-m}. \tag{3.11}$$

and $P(X_i(t) \le s_i)$ is given by (3.2).

In the general case, $F_i(K|s_i)$ can be calculated in principle, but it is a difficult calculation for large numbers of jobs and parts. [It should be noted, however, that it is still much simpler than calculating the joint probability $P(X(t) \le s)$.]

For the more difficult cases Monte Carlo provides a practical method for determining $F_i(K|s_i)$. In particular, one can generate random sequences of K jobs and simply observe the relative fraction of the sequences that do not produce stockout. Since, this is an estimate of a probability, the variance of the estimate is less than or equal to $1/4n$, where n is the number of sequences tested as noted in (2.5). This allows n to be selected to achieve the desired degree of accuracy. [Note: In Section 2, Monte Carlo was used to obtain $P(X(t) \le s)$ directly, which would be quicker than evaluating all the $F_i(K|s_i$. However, using Monte Carlo to evaluate $F_i(K|s_i)$ as discussed above provides derivative information regarding the effects of changing the stock level s_i.]

In Figure 2.2, the probability of no stockout has been plotted as a function of elapsed time for the example job matrix shown in Figure 2.1. The "Exact" and "Ind Parts" lines were discussed at the end of Section 2. The "Parts/Job" line was obtained by evaluating $F_i(K|s_i)$ for the given job matrix and then combining it with Poisson probabilities $P(M(t)=K)$ using the expansion formula. For the time interval shown here $K=6$ was sufficient to obtain three decimal place accuracy. Note that the "Parts/Job" line provides a pessimistic (lower) bound for the exact probability of no stockout, as implied by Theorem 1. In this case the bound is fairly accurate. In addition since its slope appears to approximate the exact objective function fairly well, it should serve well as a surrogate objective function. The "Parts/Job" line in Figure 2.2 is

also clearly a major improvement over the "Ind Parts" line in certain regions. For example, when the "Ind Parts" line predicts a stockout probability of 1%, the true probability is roughly 5%, while the "Parts/Job" line predicts roughly 6.5%.

4. Optimizing s

It is important to consider how $F_i(K|s_i)$ varies with s_i in order to solve optimization problems. In the case for which J_{ij} contains only 0's and 1's, it is clear, when $s_i > Kq_i$, that further increases in s_i provide decreasing marginal increases in $F_i(K|s_i)$. This is because the binomial distribution is unimodal with its maximum near Kq_i. However, in the general case, this monotonicity does not hold. This can be illustrated intuitively by considering the row $J_{i1}, \ldots, J_{im}$ and associated probabilities shown below.

# part i required	1	2	3	4
p_j	0.01	0.01	0.01	0.97

Clearly as s_i passes through each multiple of four, there will be a substantial jump in the probability of no stockout. Thus the property of decreasing marginal improvements holds only in the simpler case. Furthermore, this same type of argument can be used to show that the original objective function $P(X(t) \le s)$ is not concave in s either.

In choosing the stock level s, one would typically consider an optimization problem of the following form

$$\min_{\{s_i\}} \sum_i h_i s_i, \quad \text{subject to} \quad P\{X(t) \le s\} \ge a. \tag{4.1}$$

This problem has Lagrangian

$$\psi(s) = \sum_i h_i s_i - \mu P(X(t) \le s). \tag{4.2}$$

This Lagrangian is clearly analogous to the one that results from the problem

$$\max_{\{s_i\}} P(X(t) \le s) \quad \text{subject to} \quad \sum_i h_i s_i \le b. \tag{4.3}$$

These problems are typically not computationally feasible, because of the difficulty in evaluating the objective function. Furthermore, as noted previously, the objective functions may not be concave in s.

An alternative optimization approach for (4.3), or with the appropriate modification (4.1), is to consider meeting the lower bound on service level from Theorem 1 at minimum total inventory cost. That is, we consider the problem

$$\max_{\{s_i\}} \sum_i \log P(X_i(t) \le s_i) \quad \text{subject to} \quad \sum_i h_i s_i \le b, \tag{4.4}$$

which has Lagrangian

$$\psi(s) = \sum_i \log P(X_i(t) \le s_i) - \mu \sum_i h_i s_i \tag{4.5}$$

Problem (4.5) has the major advantage of being additively separable. In the simpler case, for which $J_{ij} = 0,1$, the individual functions $\log(P(X_i(t) \le s_i)) - \mu h_i s_i$ are concave in s_i. Let us see why this is true. First, as noted above, all the functions $F_i(K|s_i)$ experience decreasing marginal improvements as s_i is increased beyond $s_i > K\lambda_i/\lambda$. Since $P(X_i(t) \le s_i)$ is a weighted sum over K of the $F_i(K|s_i)$, where the weights are invariant with s_i, $P(X_i(t) \le s_i)$ will experience decreasing marginal improvements as well. Thus a marginal analysis approach of allocating additional stock s_i to the part that has the best improvement per unit cost will lead to a unique optimum when $J_{ij} \le 1$ for all i,j.

Example Optimization

Let consider the solution of a problem of the form (4.5) to illustrate the solution technique and the accuracy of the approximation. For this example, we will not use the job matrix in Figure 2.1, because it has entries greater than one, which would cause the individual objective functions in (4.5) to have multiple local optima. Instead, we will consider the job matrix shown below with all $J_{ij} \le 1$.

		jobs				
	1	1	1	1	1	1
	1	1	1	1	1	0
	1	1	1	1	0	0
parts	1	1	1	0	0	0
	1	1	0	0	0	0
	1	0	0	0	0	0

The holding costs (h_i) per unit will be 1,1,2,2,3,3, respectively. Jobs are assumed to be Poisson arrivals with rate $\lambda = 1$ and each job type is equally likely to occur. Thus the individual part demand rates λ_i per unit time are, respectively

1, 5/6, 2/3, 1/2, 1/3, 1/6.

We will take $t = 6$ and solve problem (4.5), with $P\{X_i(6) \leq s_i\}$ given by (3.2). Instead of choosing a particular budget constraint, we will develop the exchange curve for probability of no stockout before $t = 6$ for all inventory budgets greater than 50.

First we need to determine the form of the optimal stocking policy for (4.5) for this problem. Since we are developing an exchange curve, what we really need is the optimal order in which to stock the parts as the budget is increased. This can be determined as follows. Consider the value of μ, call it $\mu(s_i)$, such that we become indifferent between the stock level s_i and the stock level s_i+1 for part i. If we consider $\Delta\psi(s_i) = 0$ for the Lagrangian in (4.5), we obtain

$$\mu(s_i) = [\log P\{X_i(6) \leq s_i+1\} - \log P\{X_i(6) \leq s_i\}]/h_i \qquad (4.6)$$

for each part i and stock level s_i. Because of the nature of the Poisson probability in (3.2), each $\mu(s_i)$ is strictly decreasing in s_i. We can use the right hand side of (4.6) to rank the stock levels in terms of the value of μ that will cause them to first be activated. Clearly as μ decreases we pass through the various stock levels of all parts in the optimal order, since the same μ serves for all parts simultaneously. This ordering of right-hand sides of (4.6) across all part types and stock levels then gives the optimal order for successively increasing the stock levels in solving (4.5).

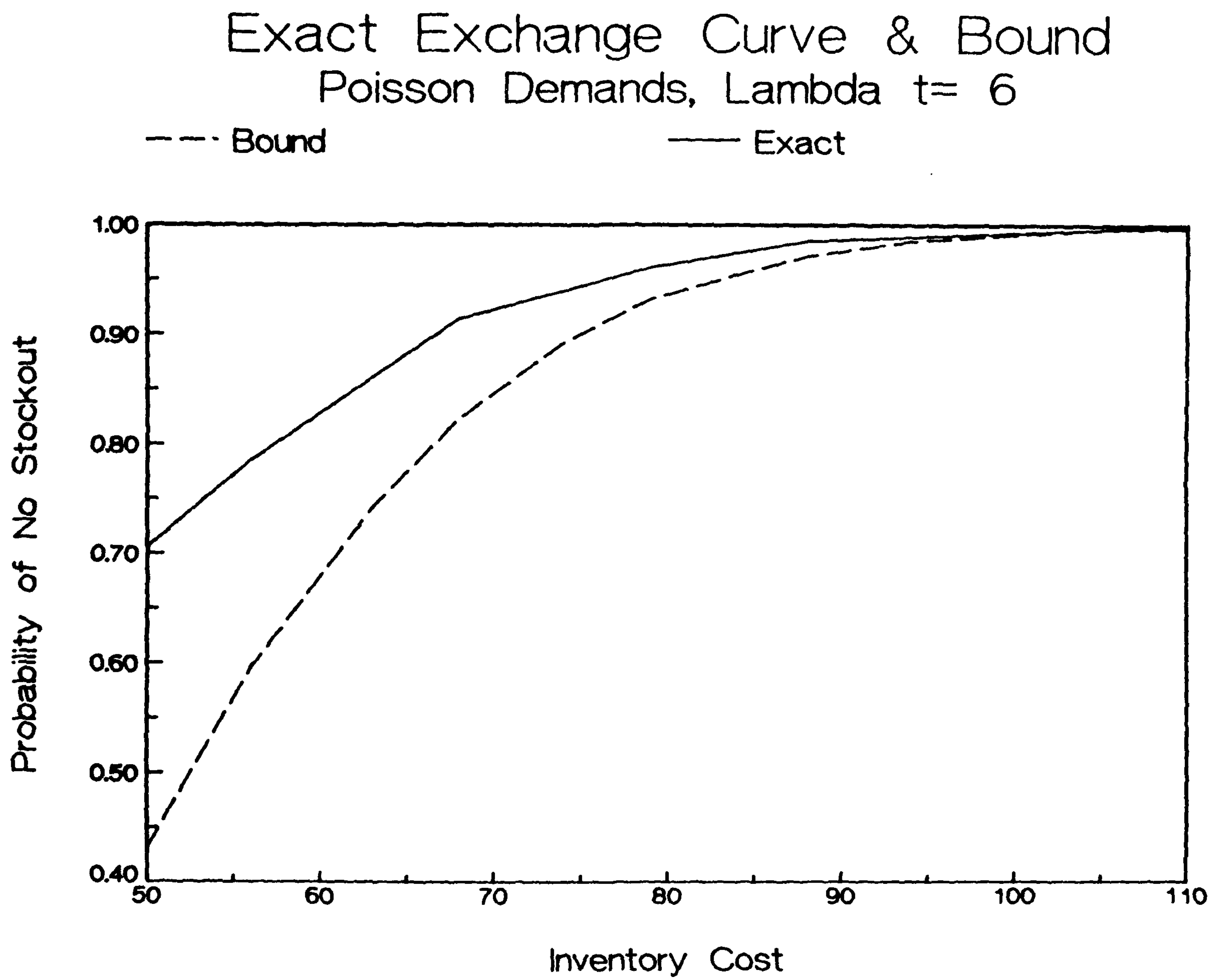
The resulting exchange curve for this problem is s' c~ in Figure 4.1. This graph was constructed from a few sample stock levels, as shown below, although all stock levels were determined in sequence for the problem.

Inventory Cost $\sum_i h_i s_i$	Stock Levels s_1	s_2	s_3	s_4	s_5	s_6
50	9	8	5	4	3	2
56	10	9	6	5	3	2
63	10	9	7	6	4	2
68	11	10	7	6	4	3
79	12	11	9	7	5	3
88	13	11	9	8	5	4
98	14	13	10	9	7	4
105	15	14	11	9	7	5

The exchange curve in Figure 4.1 shows the probability of no stockout that was achieved by each of these stock levels for the objective function in (4.3).

To test the accuracy of the bound in Theorem 1, the actual job completion probability $P(X(6) \leq s)$ was also determined for each of the stock levels s shown above by Monte Carlo. This is also plotted on Figure 4.1. [N.B. This actual probability calculation used the stock levels s shown above, which are not the optima for the actual objective function!] In this example the difference between the bound and the actual objective function is considerably larger than it was for the job matrix in Figure 2.1. This is probably because the job matrix in this section has much more commonality or linking of parts across jobs, making the independent treatment of parts a poorer assumption.

However, from an optimization standpoint, we may read the probability of no stockout from the actual curve to determine the performance that is achieved for any given inventory investment. The error introduced by our approximation of using the bound instead of the true objective function is really the difference between the performance that the inventory investment in Figure 4.1 achieves and the performance that could have been achieved by optimizing the stock level for the actual objective function subject to the



Exact Exchange Curve & Bound
Poisson Demands, Lambda t= 6
Bound
Exact
1.00
0.90
0.80
0.70
0.60
0.50
0.40
50
60
70
80
90
100
110
Probability of No Stockout
Inventory Cost

same budget. Because of the difficulties in optimizing the actual objective function, there is no way to determine precisely how large this error is. However, for any budget level, a corresponding stock level and probability of no stockout obtained from Figure 4.1, we can be certain of achieving the performance level shown with that inventory investment.

5. Conclusion

This paper considers the problem of selecting stock levels for multi-items to meet a combined objective of job completion rate or probability of filling all orders when demands for parts are interrelated. As noted by previous authors, this is both a difficult and important problem in many inventory systems. We believe that exact solutions will be computationally unfeasible for the optimization problems resulting from many applications. We have developed a lower bound function for the service level objective which is simpler to calculate. In the case of job matrices with all 0's and 1's this lower bound objective function is also relatively easy to optimize. The stock levels resulting from optimization of the lower bound can then be evaluated for the true objective function using Monte Carlo. This allows an exchange curve to be derived for the true objective function, using reasonable (although generally not optimal) stock levels. This combination of optimizing a surrogate objective function and evaluating the true objective function by *Monte Carlo* appears to be an attractive approach for a class of problems which would not be solvable otherwise.

Clearly there are important classes of problems that are beyond the scope of the solution methods described in this paper. Therefore, we believe that this is fruitful area for further research, both in the context of job completion rates for repair service systems and for order fill rates in manufacturing systems with parts commonality.

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